

## Cosmological perturbations from inhomogeneous preheating and multi-field trapping

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**ABSTRACT:** We consider inhomogeneous preheating in a multi-field trapping model. The curvature perturbation is generated by inhomogeneous preheating which induces multi-field trapping at the enhanced symmetric point (ESP), and results in fluctuation in the number of e-foldings. Instead of considering simple reheating after preheating, we consider a scenario of shoulder inflation induced by the trapping. The fluctuation in the number of e-foldings is generated during this weak inflationary period, when the additional light scalar field is trapped at the local maximum of its potential. The situation may look similar to locked or thermal inflation or even to hybrid inflation, but we will show that the present mechanism of generating the curvature perturbation is very different from these others. Unlike the conventional trapped inflationary scenario, we do not make the assumption that an ESP appears at some unstable point on the inflaton potential. This assumption is crucial in the original scenario, but it is not important in the multi-field model. We also discuss inhomogeneous preheating at late-time oscillation, in which the magnitude of the curvature fluctuation can be enhanced to accommodate low inflationary scale.

**KEYWORDS:** Cosmology of Theories beyond the SM, D-branes, Classical Theories of Gravity.

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## 1. Introduction

The primordial curvature perturbation is supposed to be generated from the perturbation of some light scalar field, whose fluctuation is generated during the primordial inflationary expansion. In the traditional inflationary scenario the inflaton potential is supposed to be responsible for both the expansion of the Universe and the generation of the curvature perturbations. Despite the simplicity of the traditional scenario, the scenario in which the inflaton is responsible both for the inflation and the curvature perturbation sometimes suffers from serious fine-tunings. This problem seems rather evident in low-scale inflationary models [1] that will be very important if some gravitational effect could be observed in LHC.<sup>1</sup> Alternatives to the traditional scenario have been discussed by many authors [5–13]. In these alternative scenarios the generation of the curvature perturbation is mostly due to the late-time conversion of the isocurvature perturbation that is related to a light scalar field other than the inflaton. The conversion mechanism characterizes the scenario. In these alternative scenarios the generation of the seed fluctuations of the light scalar field occurs during primordial inflation, and thus the typical length scale of the resultant fluctuation is very large even if the generation of the curvature perturbation occurs very late.

Among these alternatives an attractive idea of inhomogeneous reheating was considered by Dvali et al. in ref. [9] and then extended by many other authors [10], where spatial

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<sup>1</sup>In the low-scale inflationary models there are many problems related to other cosmological events. Perhaps the most significant condition appears from baryogenesis [2–4] if the scenario of the electro-weak baryogenesis fails.

fluctuations in the perturbative decay rate of the inflaton field to ordinary matter lead to fluctuations in the reheating temperature. Although this idea is very simple and is well motivated by the moduli-dependent couplings in the string model, we would like to remind the reader that in actual reheating the so-called preheating would be efficient before the conventional reheating that will be induced by the perturbative decay. Since there are various cosmological scenarios related to the preheating scenario (only some of which will be discussed in this paper), we think it is very important to construct a viable scenario for generating the curvature perturbation that works with preheating. This is our motivation to consider the scenario of inhomogeneous preheating. Besides the normal preheating that we will discuss in this paper, another possibility (instant preheating [14]) has been considered for brane inflationary model [12] and chaotic inflationary model [11, 13] including MSSM inflation [13]. As we will discuss later in this paper, our present mechanism for generating the curvature perturbations is quite different from the ones that have been based on instant preheating, although there is a similarity in the generation of the preheat field. Moreover, we are afraid that the similarity in the names “inhomogeneous reheating” and “inhomogeneous preheating” would be a confusing to the reader even though there is an essential difference between the two scenarios. In the latter scenario (inhomogeneous preheating) the perturbative decay rate is supposed to be a constant and hence it will play no role in generating the curvature perturbation, while in the former case the spatial inhomogeneity in the decay rate plays the essential role. In ref. [12] we have considered the simplest scenario of inhomogeneous preheating that induces inhomogeneous expansion during trapped inflation, which results in the fluctuation in the number of e-foldings ( $\delta N_e$ ). The essence of the inhomogeneous trapping is very different from the mechanism of inhomogeneous reheating that has been discussed in ref. [9, 10]. As we will discuss in section 3, inhomogeneous trapping is very natural in the preheating scenario. We hope the similarities in these names (“inhomogeneous reheating” and “inhomogeneous preheating”) do not create further confusion for our readers.

The idea of the present mechanism of inhomogeneous preheating is very simple. We assume that there is a light field ( $\phi_2$ ) that is accompanied by large-scale perturbations during the inflationary stage, supposing that the fluctuations related to this additional<sup>2</sup> light field can seed the fluctuations in the number density of the preheat field. During the preheating stage the kinetic energy of the inflaton ( $\phi_1$ ) is transferred into excitations of the preheat field ( $\chi$ ), which increases the actual number density of the preheat field. As we will show in section 2, the efficiency of the process can be biased by the expectation value of the secondary field ( $\phi_2$ ). Contrary to the previous analyses related to the trapping scenario, we do not assume that the ESP appears at some unstable point on the inflaton potential. In our present model the ESP is put at the minimum of the inflaton potential. Moreover, instead of considering single-field trapping, we will consider multi-field trapping in which not only the inflaton but also the secondary field is trapped at ESP, where the  $\phi_2$ -potential has a local maximum. The original assumption that the ESP appears at some unstable point on the inflaton potential could be very natural in some string models, while

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<sup>2</sup>This light field is “additional” in the sense that it is not responsible for inflation

it might be unnatural in other cosmological models. Throughout this paper, our argument does not rely on this assumption although it has played an important role in the original trapping scenario [15]. If the trapping of the field  $\phi_2$  induces another inflationary expansion, although it will be very weak compared with the primordial inflation, this inflationary stage might look very similar to “shoulder inflation” that appeared in hybrid inflationary model. The trapping mechanism might also look similar to the thermal trapping in thermal inflation. Moreover, one might think that a similar idea has been discussed in relation to locked inflation. Therefore, we think it is important to show how the present scenario differs from the others. We will shortly review these alternatives and discuss the differences in an appendix. We first describe the basic idea of inhomogeneous preheating and multi-field trapping in section 2 and 3, and then we discuss trapped inflation and its implication for the curvature perturbation in section 4. Non-gaussianity is discussed in section 5.

## 2. Inhomogeneous preheating

First we will describe the mechanism of inhomogeneous preheating with the assumption of multi-field trapping. Our discussion is based on estimates of the preheat-field production given by Kofman et al [16]. We will consider a model with real scalar fields  $\phi_i$  ( $i = 1, 2$ ) and  $\chi$ ,

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi_i + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{g^2}{2}(\phi_1^2 + \phi_2^2)\chi^2 - V_i(\phi_i), \quad (2.1)$$

where a real inflaton  $\phi_1$  and an additional light scalar  $\phi_2$  interact with a preheat field  $\chi$ . We will assume that the inflation is a chaotic type and the inflaton potential is given by

$$V_1(\phi_1) = \frac{\lambda_1|\phi_1|^{n_1}}{M_1^{n_1-4}}, \quad (2.2)$$

and the additional light field has a flat potential

$$V_2(\phi_2) = -\frac{1}{2}m^2\phi_2^2 + \frac{\lambda_2|\phi_2|^{n_2}}{M_2^{n_2-4}}, \quad (2.3)$$

which has a local maximum at the origin  $\phi_2 = 0$ . We will consider the case where the inflaton  $\phi_1$  approaches the origin and generates the preheat field  $\chi$  through preheating. The preheat field might instantly decay into light fermions by the interaction  $\sim g_\chi\chi\bar{\psi}\psi$  if the coupling  $g_\chi$  were strong, but here we assume that  $g_\chi$  is small enough to prevent the instant reheating or is even absent in the model. The mass of the preheat field  $\chi$  depends on both  $\phi_1$  and  $\phi_2$  and is given by

$$m_\chi(\phi_1, \phi_2) = g\sqrt{\phi_1^2 + \phi_2^2}. \quad (2.4)$$

Thus, immediately after the end of chaotic inflation, when  $\phi_1 \sim M_p$ , the mass of the preheat field  $m_\chi$  is very large provided that  $g$  is not extremely small. In this paper we will assume  $g \sim 1$ . Here we assume that the mass of the light scalar  $\phi_2$  is not so heavy that it cannot start oscillation immediately after inflation. The adiabatic condition is

violated when  $|\dot{m}_\chi|/m_\chi^2 \sim |\dot{\phi}_1|/g(\phi_1^2 + \phi_2^2) > 1$ , where particle production occurs. Thus the nonadiabatic condition is given by

$$|\phi_1| < \phi_1^* \equiv \sqrt{\frac{v}{g}}, \tag{2.5}$$

where  $v$  denotes the absolute value of the inflaton velocity near the origin.<sup>3</sup> Obviously the nonadiabatic region is very narrow compared with the initial amplitude of the inflaton  $\bar{\phi}_1 \sim M_p$ . As a result, we assume that the efficient particle production occurs almost instantaneously within the time interval  $\Delta t_* \sim \phi_1^*/v \sim (vg)^{-1/2}$ . Since this time interval is much smaller than the age of the Universe, one may neglect the expansion of the Universe during the particle production. Integrating over the momenta of the preheat field, the number density of the preheat field that is produced at the first impact is obtained [16],

$$n_\chi = \frac{(gv)^{3/2}}{(2\pi)^3} \exp\left(-\frac{\pi m_\chi^2}{gv}\right). \tag{2.6}$$

In the multi-field trapping, the initial value of the light field  $\phi_2$  gives the effective mass to the preheat field at the first scattering. In this case the value of the effective mass is supposed to be small so that it does not suppress the efficient preheating. The required condition is

$$\frac{\pi m_\chi^2}{gv} \simeq \frac{\pi g \phi_2^2}{v} < 1. \tag{2.7}$$

The successive scattering is discussed in ref. [16] which leads to the equations describing the occupation numbers of the preheat field with momentum  $k$  that is produced when the inflaton field passes near the ESP for  $j + 1$  times as

$$n_k^{j+1} = b_k^j n_k^j \tag{2.8}$$

where

$$b_k^j = 1 + 2e^{-\pi\mu^2} - 2 \sin \theta^j e^{-\pi\mu^2/2} \sqrt{1 + e^{-\pi\mu^2}}. \tag{2.9}$$

Here  $\mu^2 = (k^2 + m_\chi^2)/gv$ , and  $\theta^j$  is a relative phase that changes almost randomly if the time interval during each scattering is not very much shorter than the typical time scale for the change of the system parameters.

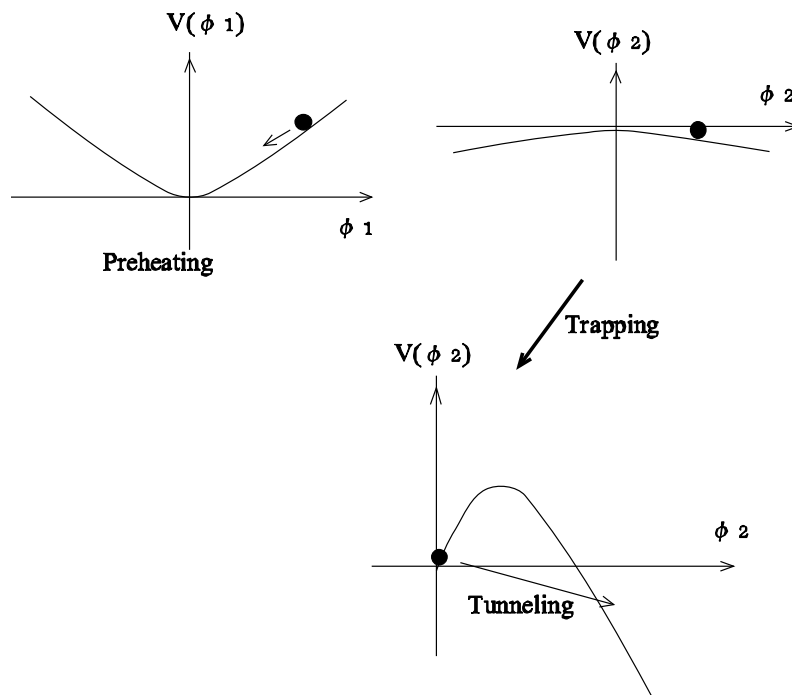
In the above calculation of the preheating, the primordial fluctuations related to the light field  $\phi_2$  appears in the fluctuation of the effective mass  $m_\chi$ .<sup>4</sup> Then the fluctuation that is produced at the first scattering is

$$\frac{\delta n_\chi}{n_\chi} \simeq \frac{2\pi g \phi_2 \delta \phi_2}{v}, \tag{2.10}$$

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<sup>3</sup>As we will discuss later in this section,  $\phi_2$  is assumed to be small so that it does not suppress the efficient production of the preheat field.

<sup>4</sup>In ref. [8], Vernizzi has discussed another scenario for generating cosmological perturbations with mass variations.



**Figure 1:** The potentials for  $\phi_1$  and  $\phi_2$  are shown in the first and the second picture. Preheating occurs due to the  $\phi_1$ -oscillation, while the trapping occurs for both fields. Since the field  $\phi_2$  gives the mass for the preheat field  $\chi$  at the minimum of the  $\phi_1$ -potential, the fluctuation  $\delta\phi_2$  will induce inhomogeneous preheating. The  $\phi_2$ -potential during trapped inflation is shown in the third picture. Since the potential barrier  $\Delta V$  decreases as  $\Delta V \propto n_\chi^2$ , trapped inflaton ends by the  $\phi_2$ -tunneling.

and the fluctuations that are generated at successive scatterings are

$$\frac{\delta b^j}{b^j} \sim \frac{2\pi g \phi_2 \delta\phi_2}{v}. \tag{2.11}$$

As we will discuss in the next section, soon after the first scattering  $\phi_2$  is trapped at the ESP and starts to oscillate with much smaller amplitude than its initial value. The value of the velocity  $v$  decreases with time because of the damping effect, but its rate of decrease cannot overcome the sudden decrease of  $\phi_2$ . In this sense the fluctuations generated from  $\delta b^j$  cannot overcome the one generated at the first scattering. As a result, the fluctuation generated at scattering is largest at the first scattering. Hence, although the scatterings occur many times during preheating, it would be fair to assume that the result is always given by eq. (2.10).

### 3. Multi-field trapping

In figure 1 we show the potentials for  $\phi_1$  and  $\phi_2$ .  $\phi_1$ -trapping and the tunneling is shown in the third picture.

The idea of the trapping mechanism at the ESP has been discussed by Kofman et al. in ref. [15]. As we have seen above, the motion of the inflaton  $\phi_1$  near the ESP

induces preheating. During preheating some of the kinetic energy of the inflaton will be transferred into excitations of the preheat field  $\chi$ , which finally leads to the number density of the preheat field  $n_\chi \sim v^{3/2}g^{-1/2}$  [15]. As the inflaton passes away from the ESP the mass of the preheat field increases, which leads to another channel for the energy transfer from the inflaton to the preheat field. In this case the energy density of the preheat field is given by

$$\rho_\chi \simeq g|\phi_1|n_\chi, \tag{3.1}$$

which grows as the inflaton goes away from the ESP. At this point the backreaction of the preheat field adds an effective confining potential to the inflaton  $\phi_1$ , and then the amplitude of the  $\phi_1$ -oscillation becomes much smaller than the original value. This is the basic idea of the trapping after preheating.

After the first scattering the light field  $\phi_2$  starts to feel the strong confining potential  $\sim g\phi_2n_\chi$ , and then starts to oscillate around the ESP with a very small amplitude. Therefore, although the roles played by the two fields ( $\phi_1$  and  $\phi_2$ ) in the inhomogeneous preheating scenario are completely different, they are both trapped just after the first scattering.

This is the idea of the multi-field trapping. We will use this idea to describe the late-time generation of the curvature perturbation.

#### 4. $\delta N_e$ from trapped inflation after inhomogeneous preheating.

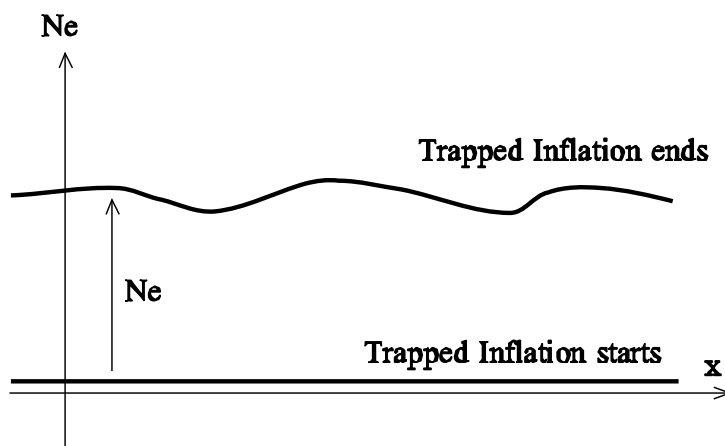
We discussed above how inhomogeneous preheating and multi-field trapping occur at the ESP. Perhaps the simplest scenario for generating the curvature perturbation with the inhomogeneous preheating would be the instant preheating scenario, in which the preheat field is supposed to decay with a homogeneous mass and a homogeneous decay rate when it (nearly) dominates the energy density of the Universe [14]. In this case the fluctuation  $\delta n_\chi$  will be converted into the fluctuation of the reheating temperature [12, 13]. Instead of considering the instant preheating scenario, here we will consider a scenario of trapped inflation that leads to the late-time generation of  $\delta N_e$ , the fluctuation of the number of e-foldings.<sup>5</sup> We think it will be straightforward to see that the typical length scale of the spatial fluctuation that is related to  $\delta N_e$  is not determined by the expansion during the trapped inflation, but is determined by the length scale of the primordial  $\phi_2$ -fluctuation that has been generated during the primordial inflation. We show a schematic picture of the  $\delta N_e$  generation in figure 2.

Now, let us consider the case where the vacuum energy around the ESP (i.e., the potential at the local maximum of  $\phi_2$ ) dominates the energy density of the Universe during some period of time after preheating. This is a multi-field version of the original idea of trapped inflation that has been discussed in ref. [15] and [12]. During trapped inflation the effective potential for the light scalar  $\phi_2$  is given by

$$V_2^{\text{eff}}(\phi_2) = V_0 - \frac{1}{2}m^2\phi_2^2 + \frac{\lambda_2|\phi_2|^{n_2}}{M_2^{n_2-4}} + gn_\chi|\phi_2|, \tag{4.1}$$

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<sup>5</sup>See also ref. [7]



**Figure 2:** The start-line of the trapped inflation is independent of the fluctuation  $\delta n_\chi$  and is given by the flat surface (the straight line at  $N_e = 0$ ). On the other hand, the end-line is determined by the number density of the preheat field  $n_\chi$ , which has the fluctuation  $\delta n_\chi$  induced by the primordial fluctuation  $\delta\phi_2$ . Note that  $\delta\phi_2$  has left the horizon during the primordial inflation.

where the constant potential  $V_0$  must be tuned so that the value of the present cosmological constant does not exceed the observational bound.<sup>6</sup> Here the mass  $M_2$  is supposed to be much larger than  $m$ . Looking at the effective potential near the origin, the effective potential for  $\phi_2 > 0$  is written as

$$V_2^{\text{eff}}(\phi_2) \simeq V_0 - \frac{1}{2}m^2 \left( \phi_2 - \frac{gn_\chi}{m^2} \right)^2 + \frac{g^2 n_\chi^2}{2m^2}. \quad (4.2)$$

Note that both  $\phi_1$  and  $\phi_2$  are strongly trapped at the origin just after the preheating because of the large number density of the preheat field ( $n_\chi \sim v^{3/2}g^{-1/2}$ ). Then the trapped inflation will be terminated when  $n_\chi$  decreases with time and finally the tunneling from  $\phi_2 = 0$  to  $\phi_2 > 2\Delta\phi_2 \equiv 2gn_\chi/m^2$  occurs. The rough estimate of the tunneling rate [18] suggests that the tunneling occurs when

$$B \sim \frac{(\Delta\phi_2)^4}{\Delta V} \sim 1, \quad (4.3)$$

where  $\Delta V$  is the height of the potential barrier,

$$\Delta V \simeq \frac{g^2 n_\chi^2}{2m^2}. \quad (4.4)$$

Therefore, the trapped inflation will be terminated when  $n_\chi$  is diluted down to  $n_\chi < m^3/g$ , where the tunneling occurs. Then the number of e-foldings elapsed during the trapped inflation is given by

$$N_e \sim \frac{1}{3} \ln \left( \frac{n_\chi(t_i)}{n_\chi(t_e)} \right). \quad (4.5)$$

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<sup>6</sup>In order to avoid the serious domain-wall problem, we need to include a  $Z_2$ -breaking parameter. The cosmological problem of the domain walls in supersymmetric model can be solved very naturally [17] if the symmetry of the domain wall is related to the R-symmetry.



In the present model,  $n_\chi(t_i)$  at the beginning of trapped inflation is supposed to be fluctuated according to eq. (2.10), while  $n_\chi(t_e)$  at the end of inflation is determined by eq. (4.3). Assuming that the primordial curvature perturbation is negligible, and also that the fluctuation related to the number of the preheat field does not induce curvature perturbation before the trapping; one can see that the curvature perturbation  $\zeta = \delta N_e \sim \frac{1}{3} \frac{\delta n_\chi(t_i)}{n_\chi(t_i)}$  is generated by trapped inflation. Since the fluctuation  $\delta N_e$  is generated due to the primordial fluctuation related to the light field  $\phi_2$ ,<sup>7</sup> the typical length scale of  $\delta N_e$  is very large compared with the expansion of the Universe due to trapped inflation. There might be another possibility that the process of the inhomogeneous preheating itself could become the origin of the curvature perturbation. It is very hard to completely exclude this possibility without using numerical calculations; however there is a reason that we can believe the generation does not occur during preheating. The reason is that during the oscillatory regime the energy density of each field will evolve as  $\sim a^{-4}$  [19] and hence the total energy density will evolve as  $a^{-4}$  irrespective of the isocurvature fluctuation related to  $\delta n_\chi$ .<sup>8</sup> Therefore, the energy density at the beginning of trapped inflation is determined by  $V_0$ , while the end-point of trapped inflation is determined by  $n_\chi(t_e) \simeq m^3/g$ . Since the isocurvature perturbation related to  $\delta n_\chi$  exists at the beginning of trapped inflation, the total  $N_e$  elapsed during trapped inflation is fluctuated as we have described above. Finally the condition for the spectrum of the perturbation is given by

$$\mathcal{P}_\zeta^{1/2} \sim \frac{g\phi_2 H_I}{v} \sim \frac{g\phi_2}{M_p} \simeq 10^{-5} \quad (4.6)$$

for  $v \simeq H_I M_p$ , which determines the initial condition for the light field  $\phi_2$  at the beginning of the preheating. Since  $\phi_2$  is bounded from above by the condition (2.7), the condition for  $H_I$  is given by

$$\sqrt{\frac{gH_I}{M_p}} > 10^{-5}. \quad (4.7)$$

It is possible to consider a hybrid-type inflationary model in which  $V_0$  dominates the energy density during the primordial inflation and hence the velocity of the inflation near the ESP will be much smaller than the above value. Introducing a small parameter  $\epsilon$  that is defined as  $v = \epsilon H_I M_p$ , we obtain for the hybrid-type inflationary model

$$\mathcal{P}_\zeta^{1/2} \sim \frac{g\phi_2 H_I}{v} \sim \frac{g\phi_2}{\epsilon M_p} \simeq 10^{-5}, \quad (4.8)$$

which leads to the condition

$$\sqrt{\frac{gH_I}{\epsilon M_p}} > 10^{-5}. \quad (4.9)$$

Besides the above lower bounds for  $H_I$ , in both cases there is an upper bound for  $H_I$  depending on the potential for the primordial inflation, since our basic requirement is

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<sup>7</sup>Note that this fluctuation is not generated during trapped inflation but has been generated during the primordial inflation

<sup>8</sup>Note that  $m_\chi \propto a^{-1}$  and  $n_\chi \propto a^{-3}$  during this regime [19].

that the the primordial curvature perturbation that could be generated by the traditional inflationary mechanism does not dominate the cosmological perturbation. For example, in chaotic inflationary model with quadratic potential, we obtain the bound

$$10^{13} GeV > H_I > \frac{\epsilon}{g} 10^8 GeV, \tag{4.10}$$

which suggests that small coupling constant  $g < \epsilon 10^{-5}$  is not acceptable in our model.

### 5. Inhomogeneous preheating and late-time oscillation

Inhomogeneous preheating may occur whenever oscillation starts during the evolution of the Universe. Here we will consider the case where  $\phi_1$  is a light field that starts to oscillate late after the inflaton oscillation but before the  $\phi_2$ -oscillation, taking that the  $\phi_1$ -velocity at the first scattering at the ESP is given by  $v \simeq H(t_{osc})\phi_1(t_{osc})$ , where  $t_{osc}$  is the time when  $\phi_1$  starts to oscillate. Then what happens after the first scattering is almost the same as in the previous scenario provided that the other light field  $\phi_2$  satisfies the required condition  $g\phi_2^2/v < 1$ . The trapped inflation starts when  $V_0$  starts to dominate the energy density of the Universe, and ends when  $n_\chi$  decreases to  $n_\chi \simeq m^3/g$ . Here the form of the  $\phi_2$ -potential is supposed to be the same as the previous one.

The number density of the preheat field generated by the preheating is  $n_\chi \sim v^{3/2}g^{-1/2}$  [15]. If the energy density of the Universe evolves as  $\propto a^{-4}$  before trapped inflation,  $n_\chi$  at the beginning of the trapped inflation is

$$n_\chi \sim \frac{v^{3/2}}{g^{1/2}} \times \left( \frac{V_0}{H^2(t_{osc})M_p^2} \right)^{3/4}. \tag{5.1}$$

Demanding that the number density of the preheat field at the beginning of trapped inflation is larger than  $n_\chi \simeq m^3/g$ , we find the condition

$$\phi_1 > \frac{m^2 M_p}{g^{1/3} V_0^{1/2}}. \tag{5.2}$$

The condition for the spectrum of the perturbation is given by

$$\mathcal{P}_\zeta^{1/2} \sim \frac{g\phi_2 H_I}{v} \simeq \frac{g\phi_2 H_I}{H_{osc}\phi_1} \simeq 10^{-5}, \tag{5.3}$$

which is a much looser condition than the one obtained in the previous scenario because of the possible hierarchy  $H_I/H_{osc} \gg 1$  and  $\phi_1/M_p \ll 1$ . The situation looks very similar to the curvaton paradigm in which the additional phase transition accommodates low inflationary scale [6]. Since the dimensionless parameter  $\epsilon$  is now given by

$$\epsilon = H_{osc}\phi_1/H_I M_p, \tag{5.4}$$

the condition (4.9) obtained in the previous section leads to

$$H_I > 10^{-5} \sqrt{\frac{H_{osc}\phi_1}{g}}. \tag{5.5}$$

## 6. Non-gaussianity

In the above analysis we neglected higher terms that are proportional to  $(\delta\phi_2)^n$  ( $n \geq 2$ ). This approximation is not appropriate if the impact parameter is small while the fluctuation of the impact parameter is large. Therefore, we will examine the non-gaussianity condition for more details and see how we can estimate the non-gaussianity parameter  $f_{NL}$ . The value of the non-gaussianity parameter  $f_{NL}$  is determined by the non-gaussian contribution to the Bardeen potential

$$\Phi = \Phi_G + f_{NL}\Phi_G^2, \tag{6.1}$$

where  $\Phi_G$  denotes the gaussian part. The relation between  $\Phi$  and  $\zeta$  is given by

$$\Phi = -\frac{3}{5}\zeta = -\frac{1}{5}\frac{\delta n_\chi}{n_\chi}. \tag{6.2}$$

The expression for  $\delta n_\chi/n_\chi$ , which is approximately given by eq. (2.10) contains higher terms that are proportional to  $(\delta\phi_2)^2$ . The expression for  $\delta n_\chi/n_\chi$  up to the second order is given by

$$\frac{\delta n_\chi}{n_\chi} \simeq -\frac{2\pi g\phi_2\delta\phi_2}{v} - \frac{1}{2}\left(\frac{2\pi g}{v} - \frac{4\pi^2 g^2\phi_2^2}{v^2}\right)(\delta\phi_2)^2. \tag{6.3}$$

Therefore, the non-gaussianity parameter is

$$-\frac{3}{5}f_{NL} \simeq \frac{3v}{4\pi g\phi_2^2} - \frac{3}{2}, \tag{6.4}$$

where the factor of  $3/2 > 1$  appears. Therefore, unlike the traditional inflationary scenario the non-gaussianity parameter  $|f_{NL}|$  in our present model is always<sup>9</sup> larger than unity.

## 7. Conclusions and discussions

In this paper we described an alternative to the traditional inflationary scenario using the ideas of (1) multi-field trapping, (2) inhomogeneous preheating and (3) trapped inflation. As we have discussed in ref. [12], it is straightforward to apply this mechanism to the brane inflationary model. The curvature perturbation is generated not by the primordial inflation but by the trapped inflation, while the isocurvature fluctuation that seeds the generation of the curvature perturbation at a later stage is generated during primordial inflation. In the present case the non-gaussianity parameter  $f_{NL}$  is always larger than unity. Such a large non-gaussianity is a distinguishable feature of the model.

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<sup>9</sup>Here we do not consider fine-tunings that may result in the cancellation between the two terms.

## A. Other models

- Locked Inflation

Note that if there were direct coupling  $\sim g^2\phi_1^2\phi_2^2$ , the model turns into a conventional hybrid scenario. Then the light scalar  $\phi_2$  will be trapped at the origin during inflation and also during the inflaton oscillation. This is a scenario called “locked inflation” that occurs after hybrid inflation, and would be very important if one is considering a hybrid inflationary model. However, in this scenario one cannot generate the curvature perturbation from the fluctuation related to the light scalar  $\phi_2$ , simply because the light field grows to a huge mass  $\sim g\phi_1$  during inflation.

- Thermal inflation

We will also comment on the thermal inflationary model, which has a similar flat potential and is also based on the similar idea that a light scalar is trapped at the origin to trigger weak inflation. Obviously the crucial difference between the two scenarios is in the mechanism of the trapping. In our model we considered multi-field trapping<sup>10</sup> due to the confining force induced by the preheat field, instead of considering the thermal effective potential.

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<sup>10</sup>We use the word “multi-field” in the sense that both inflaton  $\phi_1$  and light scalar  $\phi_2$  are trapped at ESP

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